angle of attack is greatest on the swept edges and windward surface. The results of this study show that these areas can be sheathed by a layer of cool gas supplied by windward injection.

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A New Guidance System Figure-of-Merit

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A CONVENIENT parameter to determine the relative effectiveness of a launch vehicle in delivering a spacecraft onto an interplanetary trajectory and to determine midcourse velocity budgets is the expected value of the midcoursec orrection velocity magnitude v. The parameter in common use, while it is sometimes called the expected value of v, is the square root of the trace of the midcourse velocity covariance matrix.[1]

The figure-of-merit, defined in this manner, is often used to approximate midcourse correction velocity budgets by using the further approximation[2]

$$\mu|\bar{v}| + 3\sigma|\bar{v}| \simeq 3\text{FOM} \quad (1)$$

where $|\bar{v}| = v = (v_1^2 + v_2^2 + v_3^2)^{1/2}$, $\mu|\bar{v}|$ is the mean of v, and $\sigma|\bar{v}|$ is the standard deviation of v.

This approximation is equivalent to assuming that the mean magnitude is zero (which would be true for a linear process with the means of the components zero) and, therefore, that the second moment of the magnitude about zero is the standard deviation.

Using Eq. (1) the probability that v does not exceed 3FOM is greater than or equal to 99%

$$P[v \leq 3\text{FOM}] \geq 0.99 \quad (2)$$

where v_1,v_2,v_3 = components of $\bar{v}$ in any Cartesian coordinate system.

It is the purpose of this Note to suggest a new FOM, for which Eq. (2) is still true, that saves between 13 and 30% of the v budget. The new approximation has one other, important, characteristic. It distinguishes between two midcourse velocity matrices with the same trace but with different midcourse velocity requirements. Since the old FOM is used extensively to compare the midcourse requirements for different trajectories, this is a serious deficiency that can be eliminated by the new approximation.

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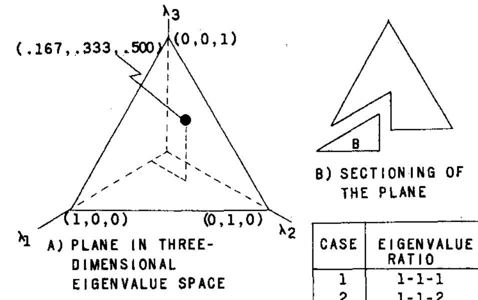

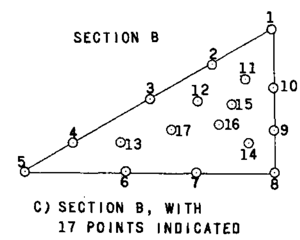

CASE	EIGENVALUE RATIO
1	1-1-1
2	1-1-2
3	1-1-4
4	1-1-13
5	1-0-0
6	1-4-0
7	7-13-0
8	1-1-0
9	2-9-9
10	1-2-2
11	443-333-224
12	57-27-16
13	1-2-10
14	52-41-7
15	1-2-3
16	1-3-5
17	2-5-13

Fig. 1 Definition of eigenvalue ratios.

This new FOM is based on an analysis by Hoffman and Young[3] who obtained an approximation to the mean and variance of v by expanding v^n in a Taylor series about a point in a transformed space defined by

$$v_1^* = |v_1|, \qquad v_2^* = |v_2|, \qquad v_3^* = |v_3| \quad (3)$$

truncating after the second-order terms and integrating over the transformed density function to define the nth moment of v. The components v_1, v_2, and v_3 were assumed to be normally distributed with mean zero. The analysis resulted in the approximations

$$\mu|\bar{v}| = E(|\bar{v}|) \simeq (2T/\pi)^{1/2}[1 + (\pi - 2)\beta/(2A)^{1/2}T^2] \quad (4a)$$

$$\sigma|\bar{v}|^2 \simeq T - \{(2T/\pi)^{1/2}[1 + (\pi - 2)\beta/(2A)^{1/2}T^2]\}^2 \quad (4b)$$

where $\lambda_1,\lambda_2,\lambda_3$ = eigenvalues of the v matrix, T = trace of the v matrix = $\lambda_1 + \lambda_2 + \lambda_3$, $|\bar{v}|$ = magnitude of the v vector $(v_1,v_2,v)^T$, A = constant used to find the best point about which to expand, $E[\ ]$ = expected value operator, and $\beta = \lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3$. The series expansion was made about the point $[(A\lambda_1/\pi)^{1/2},(A\lambda_2/\pi)^{1/2},(A\lambda_3/\pi)^{1/2}]$. The maximum percentage error in the variance of v was minimized resulting in a value of 2.7 for A.

The new FOM is taken to be

$$\text{FOM} = \mu|\bar{v}| + 3\sigma|\bar{v}| \quad (5)$$

with the mean given by Eq. (4a) and the standard deviation by Eq. (4b). To check the accuracy for this new FOM 17 covariance matrices were chosen to be representative of general midcourse correction covariance matrices. Each of the matrices was chosen such that $T = 1$ and the eigenvalues had the various ratios defined in Fig. 1.

In Fig. 1a is presented a plane in three-dimensional eigenvalue space equivalent to $T = 1$. Every choice of three eigenvalues (with $T = 1$) must be a point on the plane. In Fig. 1b is a small sketch indicating that the eigenvalue plane has been sectioned, and Sec. B is presented in Fig. 1c. The 17 points are indicated along with the corresponding eigenvalue ratios. Because of symmetry of the mathematical formulation, each point of Sec. B is equivalent to a point on each of the remaining five sections. Investigating the results for these 17 cases is equivalent to considering 76 points in the eigenvalue plane which should be more than adequate.

Table 1 Accuracy of the new FOM for various eigenvalue ratios

[A = 2.7]

1	2	3	4	5	6	7	8	9	10	11
EIGENVALUES			EXACT MEAN	APPROXIMATE MEAN	EXACT STANDARD DEVIATION	APPROXIMATE STANDARD DEVIATION	EXACT VALUE FOM	APPROX. VALUE, FOM	% ERROR	$P[\lvert\bar{V}\rvert \le \mu_{\lvert\bar{V}\rvert} + 3\sigma_{\lvert\bar{V}\rvert}]$
λ_1	λ_2	λ_3								
0.333	0.333	0.333	0.9213	0.9285	0.3888	0.3712	2.087	2.042	-2.15	99.4
.250	.250	.500	.9158	.9204	.4016	.3910	2.121	2.093	-1.32	99.0
.666	.166	.166	.8991	.8958	.437	.444	2.210	2.228	+0.815	99.3
.866	.066	.066	.8591	.8449	.5118	.5349	2.394	2.449	+2.30	99.1
1.00	0	0	.7978	.7978	.6028	.6028	2.606	2.6062	+0.008	99.0
.800	.200	0	.8643	.8606	.5030	.5093	2.373	2.388	+0.630	99.0
.650	.350	0	.8811	.8870	.4728	.4616	2.299	2.2718	-1.180	99.2
.500	.500	0	.8862	.8958	.4632	.4443	2.276	2.228	-2.11	99.3
.450	.450	.100	.9077	.9125	.4195	.4089	2.166	2.139	-1.25	99.2
.400	.400	.200	.9173	.9233	.3981	.3840	2.112	2.075	-1.76	99.4
.443	.333	.224	.9179	.9238	.3967	.3827	2.108	2.073	-1.67	99.4
.570	.270	.160	.9089	.9109	.4168	.4126	2.159	2.149	-0.47	99.3
.769	.154	.0769	.8809	.8721	.4730	.489	2.300	2.339	+1.69	99.2
.520	.410	.070	.9025	.9069	.4306	.4212	2.194	2.1705	-1.08	99.4
.500	.333	.166	.9133	.9176	.4074	.3974	2.135	2.110	-1.18	99.1
.555	.333	.111	.9063	.9092	.4225	.4164	2.174	2.158	-0.74	99.0
.650	.250	.100	.8984	.8968	.4392	.4423	2.216	2.224	+0.36	99.5

* OLD FOM IS 1.0

For each of these 17 cases the mean and standard deviation were obtained from Eqs. (4a) and (4b), respectively, and are presented in columns 5 and 7 of Table 1. The exact values of columns 4 and 6 were calculated by multiplying the probability density function of $\bar{v}$ by $|\bar{v}|$ and $|\bar{v}|$ squared and performing a numerical integration. The integrated values were accurate to 12 places.

The new exact figure-of-merit is presented in column 8. It can easily be seen that these values are in most cases considerably less than 3 (i.e., 3 times the old FOM). The new approximate FOM is presented in column 9 and the error in column 10. The error in the new FOM is in no considered case greater than 2.3%. In column 11 is presented the probability that v will not exceed the new FOM. These probabilities were determined from the numerical integration.

Use of the new FOM requires knowledge of not only the trace of the midcourse correction velocity, but also the quantity β. Since this quantity is a function of the eigenvalues of the covariance matrix a diagonalization of the matrix would normally be required. However, it is shown in an appendix to Ref. 3 that β can be written

$$\beta = E(v_1'v_1')E(v_2'v_2') + E(v_1'v_1')E(v_3'v_3') + E(v_2'v_2')E(v_3'v_3') - E^2(v_2'v_3') - E^2(v_1'v_2') - E^2(v_1'v_3') \quad (6)$$

where the prime denotes any cartesian coordinate system. Thus, by the use of Eqs. (4a), (4b), and (6), the new FOM for any covariance matrix can be found by only algebraic manipulations.

In conclusion, a new figure-of-merit has been defined which is more accurate than the one presently used and can be calculated quickly. The new approximation saves midcourse correction velocity and distinguishes between covariance matrices with the same trace but with different eigenvalue ratios.

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